Math 1B Quiz 1 Version 1
Mon Jan 23, 2017

NAME YOU ASKED TO BE CALLED IN CLASS: [redacted]

SCORE: _____ / 30 POINTS +1 ← GREENSHEET QUIZ

1. NO CALCULATORS OR NOTES ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

A person's velocity (in meters per minute) at time t (in minutes) is given by $v(t) = \begin{cases} 20-2t, & 0 \le t \le 8 \\ t-4, & 8 \le t \le 18 \end{cases}$. **SCORE: _____ / 5 PTS**

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds.
NOTE: You must show the arithmetic expression that you used to get your answer.

$s(18) = \frac{v(0)+v(8)}{2}(8) + \frac{v(18)+v(8)}{2}(10)$

$= \underbrace{\frac{4+18}{2}(8)}_{*} + \underbrace{\frac{14+4}{2}(10)}_{1} = 11(8) + 9(10) = 88 + 90$

$\underbrace{= 178\text{ m}}_{*}$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds using three subintervals and left endpoints.
NOTE: You must show the arithmetic expression that you used to get your answer.

$\sum_{i=0}^{2} f(i\Delta x)\Delta x \; ; \; \Delta x = \frac{18-0}{3} = 6$, $f(0)\cdot 6 + f(6)\cdot 6 + f(12)\cdot 6$

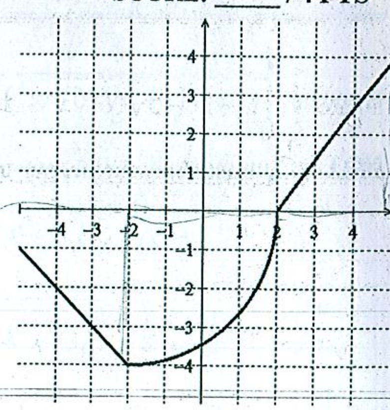
$\underbrace{20\cdot 6 + 8\cdot 6 + 8\cdot 6}_{2}$

$120 + 48 + 48$

$120 + 96$

$\underbrace{\approx 216\text{ m}}_{\frac{1}{2}}$

The graph of function f is shown on the right. **SCORE: _____ / 4 PTS**
The graph consists of a diagonal line, an arc of a circle, then another diagonal line.



[a] Evaluate $\int_{-5}^{5} f(x)\,dx$.

NOTE: You must show the arithmetic expression that you used to get your answer.

$\int_{-5}^{5} f(x)\,dx = \underbrace{-\left(\frac{(4+1)}{2}(4)\right)}_{*} - \underbrace{\frac{\pi 4^2}{4}}_{1} + \underbrace{\frac{3(4)}{2}}_{\frac{1}{2}}$

$= -\left(\frac{20}{2}\right) - 4\pi + \frac{12}{2} = -10 - 4\pi + 6 \,\Big|\, \underbrace{= -4 - 4\pi}_{*}$

[b] Evaluate $\int_{5}^{-2} f(x)\,dx$.

$\int_{5}^{-2} f(x)\,dx = -\int_{-2}^{5} f(x)\,dx = -\left(\frac{\pi 4^2}{4}\right) + \frac{3(4)}{2} \boxed{= 6 - 4\pi}$

. **Using the limit definition of the definite integral, and right endpoints,** find $\int_{-3}^{-1}(3x^2+15x+18)\,dx$. **SCORE: _____ / 10 PTS**

NOTE: Solutions using any other method will earn 0 points.

$\Delta x = \frac{b-a}{n} = \frac{-1-(-3)}{n} = \frac{2}{n}$

$\int_{-3}^{-1}(3x^2+15x+18)\,dx$

$= \lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\,\Delta x \quad ; \quad a=-1,\ \Delta x = \frac{2}{n}$ 1

$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{2i}{n}\right)\frac{2}{n}$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n} 3\left(-1+\frac{2i}{n}\right)^2+15\left(-1+\frac{2i}{n}\right)+18$

1 ✱

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n} 3\left(\frac{4i^2}{n^2}-\frac{4i}{n}+1\right)-15+\frac{30i}{n}+18$

$= \lim_{n\to\infty}\frac{6}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2}-\frac{4i}{n}+1-5+\frac{10i}{n}+6$

$= \lim_{n\to\infty}\frac{6}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2}+\frac{6i}{n}+2$ ✱

$= \lim_{n\to\infty}\frac{12}{n}\sum_{i=1}^{n}\frac{2i^2}{n^2}+\frac{3i}{n}+1$

$= \lim_{n\to\infty}\frac{12}{n}\left(\sum_{i=1}^{n}\frac{2i^2}{n^2}+\sum_{i=1}^{n}\frac{3i}{n}+\sum_{i=1}^{n}1\right)$

$\lim_{n\to\infty}\frac{12}{n}\left(\frac{2}{n^2}\sum_{i=1}^{n}i^2+\frac{3}{n}\sum_{i=1}^{n}i+\sum_{i=1}^{n}1\right)$

$\lim_{n\to\infty}\frac{12}{n}\left(\frac{2}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)+\frac{3}{n}\left(\frac{n(n+1)}{2}\right)+n\right)$ 1 1

$\lim_{n\to\infty}12\left(\frac{2n(n+1)(2n+1)}{6n^3}+\frac{3n(n+1)}{2n^2}+1\right)$

$= 12\left(\frac{4}{6}+\frac{3}{2}+1\right)$

$= 12\left(\frac{4}{6}+\frac{9}{6}+\frac{6}{6}\right)$

$= 12\left(\frac{19}{6}\right) = \frac{91}{6}$

1 LIM ON EACH LINE WITH "n"
$n\to\infty$

Evaluate $\int_{-4}^{4}(|x-1|-5\sqrt{16-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: 2 / 5 PTS**

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$\int_{-4}^{4}|x-1| - 5\int_{-4}^{4}\sqrt{16-x^2}$ 2 FORGOT dx ?

$\int_{-4}^{4}|x-1|-5\sqrt{\int_{-4}^{4}16-x^2}$

$\int_{-4}^{4}|x-1|-5\sqrt{\int_{-4}^{4}x+4\cdot\int_{-4}^{4}x-4}$

$\int_{1}^{4}|x-1|+\int_{-4}^{1}|x-1|-5\sqrt{\int_{-4}^{4}x+4\cdot\int_{-4}^{4}x-4}$

$\frac{|4-1|+|1-1|}{2}(3)+$